DYNAMIC STABILIZATION OF THE INTERCHANGE INSTABILITY OF A LIQUID-GAS INTERFACE*

G. H. Wolf

Institut für Plasmaphysik GmbH, Garching bei München, Germany

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The Rayleigh-Taylor instability of a viscous liquid superimposed upon air can be dynamically stabilized by oscillating the liquid perpendicularly to its horizontal equilibrium surface, thus maintaining the position of the liquid for an arbitrary time. The viscosity of the liquid was found to have a strong influence on the stability of the short-wavelength modes. In the parameter regime investigated the effect of the compressibility of the gas was negligible.

Recently it was shown that the Rayleigh-Taylor instability[1-6] of superimposed liquids can be dynamically stabilized[7] by oscillating the liquids perpendicularly to their horizontal equilibrium interface, provided the viscosity of one of the liquids is sufficient to suppress the development of "parametric resonances" due to otherwise rapidly growing short-wavelength modes. When this experiment is considered as a model case for stabilization problems of magnetically confined plasmas, two major differences between the two situations become apparent: First, the boundary between the plasma and the magnetic field is a diffuse rather than a sharp one and secondly, the compressibility of plasmas is similar to that of gases rather than that of liquids.

As to the diffuse boundary layer it could be shown both theoretically[8] and experimentally[9] that dynamically such a system cannot be completely stabilized; although gross stability is achieved and "parametric resonances" can be suppressed even for negligible viscosity,[9] the modes with short vertical wavelengths (in a plasma suppressed by finite-Larmor-radius effects![8]) can have eigenfrequencies small enough to violate the stabilization condition for any applied oscillation energy. These unstabilized modes result in density fluctuations leading to a continuous rearrangement of the boundary sheath and to some kind of enhanced diffusion; experimental results on these processes are described elsewhere.[9]

In the following, an experiment is described where one of the two liquids is replaced by air. This allows us to look for the possible influence of the compressibility on the dynamic stabilization condition. For the case of two liquids the stabilization condition has already been derived.[7] Neglecting viscosity it can be written as

$$b_m/g \gtrsim \sqrt{2}\omega/\Omega_k, \tag{1}$$

where b_m is the maximum instantaneous acceleration due to the enforced harmonic oscillation of frequency ω, g is the acceleration of gravity, and Ω_k are the eigenfrequencies (growth rates) of possible standing surface-wave modes with wavelength L_k. If the difference in density of the two liquids is large, then[1]

$$\Omega_k^2 = 2\pi g/L_k. \tag{2}$$

Inserting into Eq. (1) the minimum eigenfrequency Ω_0 possible in a cylindrical vessel of diameter D, one obtains[7]

$$b_m/g \gtrsim \omega(0.54D/g)^{1/2}. \tag{3}$$

There is a second limit of the stability region due to the fastest growing modes. It can be expressed by

$$b_m/g \leq F(\omega), \tag{4}$$

where $F(\omega)$ depends on the growth rates of these modes, which are determined by the viscosity or surface tension[4] of the liquid. Violating condition (4) excites a type of instability, which we call "parametric resonance" by extending the usual meaning[10] of this expression.

Experimentally, liquids of various viscosity (in turn liquid paraffin and motor oils), but of about equal (negligible) surface tension were used. A cylindrical vessel of inner diameter $D = 1.6$ cm was mounted vertically upon a vibrator which could produce values of b_m/g up to 60. The vessel was filled with the particular liquid up to a depth of about 3 cm. Then the vibrator was adjusted to vertical oscillations ($b_m/g = 60$; $\omega/2\pi = 200$ Hz). As described previously,[7] the whole system was then turned upside down, thus passing through dynamic equilibrium states[7] of the liquid-air interface. The vessel could be kept completely open at the side of the gaseous medium, although it was usually closed to prevent the liquid from dropping out when the oscillation parameters fell short of condition (3) or (4). While violating condition (3) resulted in some gross slipping of the liquid down the walls of the tube, violating condition (4) excited crispations of the surface which were eventually transformed into falling rainlike drops.

Figure 1 shows a plot of the experimental results. The experimental error is not known precisely, partly owing to deviations of the applied oscillation wave form from a true sine wave. These might be responsible, for instance, for the slight modulation of the experimental points around curve A (see below). Another source of experimental error is ascribed to wall effects, depending on whether the vessel wall in the neighborhood of the stabilized surface had already been wetted by the liquid.

For a given viscosity each series of values of b_m/g vs ω consists of two branches which demarcate the stability region. The upper branches, which are related to condition (4), show a strong dependence on the viscosity, while the lower branches, which are related to condition (3), coincide more or less, showing that the latter condition is independent of viscosity over a wide range. This result can be qualitatively explained by the strongly increasing influence of the viscosity on the short-wavelength modes[4] determining condition (4). It should be mentioned, however, that for the liquid with the highest viscosity (6.18 P) even stabilization due to Eq. (3) could not be achieved below $b_m/g \approx 35$. For this case and below this limit a slowly flowing liquid channel was formed from the surface downwards along the vessel wall. We are inclined to attribute this effect also to the influence of the viscosity, which prevents the liquid near the wall from taking part in the necessary oscillatory motion.

While the dashed line A in Fig. 1 is the theoretical curve corresponding to Eq. (3), the solid lines a-d are the graphical representation of the upper limit of an empirically found equation describing the viscosity dependence of the parametric resonances: By adjusting the right-hand side of Eq. (4) to fit the experimental values shown in Fig. 1 one obtains

$$b_m/g \lesssim 0.5\eta^{2/5}\omega/2\pi - 30, \tag{5}$$

where η has to be expressed in poise. Extrapolating Eq. (5) to values of η much lower than those used experimentally gives a rough estimate of a critical value η_c below which the conditions (3) and (5) should no longer be satisfied simultaneously, i.e., viscosity damping alone should not be sufficient to suppress parametric resonances when operating above line A of Fig. 1. For the vessel diameter of our experiment η_c is of the order of 0.1 P, which is ten times the viscosity

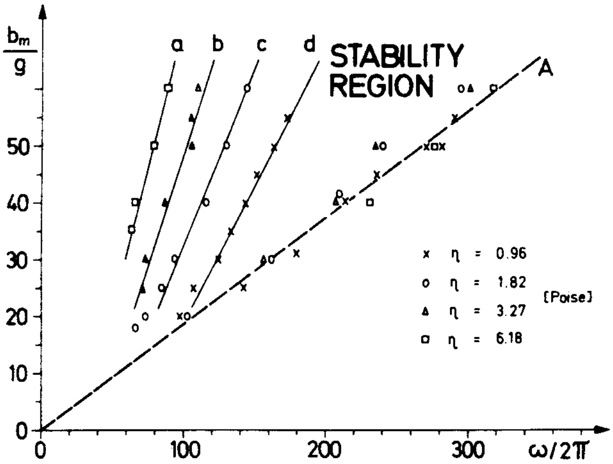

FIG. 1. Plot of the experimental results showing the lower and upper limit of the stable region. The theoretical lower limit is indicated by the dashed line A following from Eq. (3) for $D = 1.6$ cm. The solid lines a-d represent the empirical formula (5) for $\eta = 6.18$, 3.27, 1.28, and 0.96 P, respectively.

of water. Assuming that the above results on parametric resonances are not essentially dependent on the particular vessel diameter leads to the more general expression

$$\eta_c \approx 0.05 D^{5/4} \tag{6}$$

(η in poise, D in centimeters). Consequently for η_c to reach the viscosity of water D approaches the order of a quarter of a centimeter, which is so small that surface tension and wall effects (e.g., interfacial tension) are no longer negligible.

In order to transform the representation of the parametric resonances from Fig. 1 to Mathieu's stability chart,[10] one has to know the undamped eigenfrequency Ω_p of the parametrically excited mode as a function of ω, which can be done by measuring the excited wavelength $L_p(\omega)$. This is the subject of a more detailed study.[11]

As the main experimental result it can be stated that in the parameter regime covered experimentally the stabilization condition (3), which was derived for a liquid-liquid interface neglecting compressibility, also fits for a liquid-gas interface. In order to explain this coincidence let us discuss possible mechanisms due to compressibility which might occur when replacing one of the liquids by a gas.

(1) The instantaneous maximum hydrostatic pressure p_m (g replaced by b_m) of the superposed liquid may exceed the gas pressure. This was not the case in our experiment, where p_m

≈ 0.2 atm.

(2) If the lower boundary of an enclosed gas volume is not oscillating together with the system, the free surface of the liquid acting as a piston can excite standing pressure waves when $2\pi c_s/\omega \gtrsim \Lambda$ where c_s is the sound speed of the gas and Λ is the axial length of the gas volume. This was not the case in the present experiment either.

(3) The essential influence of the compressibility occurs when the instantaneous instability growth time $g/b_m\Omega_k$ becomes so small that it approaches the sound transit time $L_k/2c_s$ or D/c_s. Using Eq. (2) gives a rough condition for this situation,

$$b_m/g \to 2c_s(\pi g L_k)^{-1/2}. \tag{7}$$

It follows from Eq. (7) that in our experiment ($L_k \lesssim 2D$) even when using Frigen instead of air values of b_m/g of some hundreds would be necessary in order to operate in a compressibility-dominated regime. Although the vibrator available for the present experiment did not allow the values of b_m/g to approach condition (7), this should be possible by using a high-power loudspeaker. Operating in the regime stipulated by Eq. (7) would allow the gas pressure to vary locally across the flute perturbations of the equilibrium surface. Since one might expect that the relatively increased (decreased) gas pressure acting on the flutes in the regions of higher (lower) hydrostatic pressure constitutes an additional stabilizing term, an investigation of this situation would be of interest.

The discussion has thus shown that the results of Fig. 1 should indeed coincide with the line A derived for incompressible liquids. Concerning the compressibility effects, therefore, the present experiment shows analog features only to those dynamic stabilization and equilibrium schemes[12-14] for confined plasmas where (a) the applied oscillation time is long compared with the characteristic magnetoacoustic transit time, for instance, in the radial direction and where (b) the instantaneous instability growth time is long compared with the magnetoacoustic transit time across a virtual flute perturbation.

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